## Supplementary Material C
## Expected decision time and probability of action

We let $T_c$ denote the time to the cortex deciding upon anti-predator action. Often, this will take no value, as action will not always occur in cases with negative drift (when there is no predator).

To calculate $E(T_c|\mu_-, \text{AP action})$, we can make use of the survival rate calculation that was employed when determining the optimal boundary position in Supplementary Material B. In this case, rather than analysing the animal's probability of survival, $f(z)$, we let $\phi(z)$ denote the probability of the Brownian motion process surviving, again with probability 1 if it starts at the boundary and allowing the expected probability of survival to decrease with time according to $E(e^{-\lambda T})$ for some $\lambda$, so that $\phi(z) \to 0$ as $z$ tends to $-\infty$. Note that, unlike $\theta$, we are free to choose $\lambda$ independently of the animal's survival rate.

As we have the same equations (including boundary conditions) as when calculating the probability of animal survival, the probability of the process surviving is

$$\phi(0) = E(e^{-\lambda T}|Z_0 = 0) = \alpha^j \tag{1}$$

where the boundary is at $-ln\alpha$ and $j = \frac{-\mu+\sqrt{\mu^2+2\lambda\eta^2}}{\eta^2}$.

Differentiating with respect to $\lambda$ and evaluating at $\lambda = 0$, from the left hand side of equation (1), we get:

$$\frac{\partial}{\partial\lambda}E(e^{-\lambda T})|_{\lambda=0} \;=\; E\left(\frac{\partial}{\partial\lambda}e^{-\lambda T}\right)\bigg|_{\lambda=0} \;=\; E(-Te^{-\lambda T})|_{\lambda=0} \;=\; -E(T).$$

From the right hand side of equation (1), we have:

$$\frac{\partial}{\partial\lambda}\alpha^{\frac{-\mu+\sqrt{\mu^2+2\lambda\eta^2}}{\eta^2}}\Bigg|_{\lambda=0} \;=\; \frac{\alpha^{\frac{-\mu+\sqrt{\mu^2+2\lambda\eta^2}}{\eta^2}}ln(\alpha)}{\sqrt{\mu^2+2\lambda\eta^2}}\Bigg|_{\lambda=0} \;=\; \frac{\alpha^{\frac{-\mu+|\mu|}{\eta^2}}ln(\alpha)}{|\mu|}.$$

Thus,

$$E(T|\text{AP action}) = \frac{\alpha^{\frac{-\mu+|\mu|}{\eta^2}}ln(\alpha)}{|\mu|}.$$

As $\mu = \mu_- < 0$ when no predator is present,

$$E(T_c|\mu_-, \text{AP action}) = \frac{ln(\alpha)}{\mu_-\alpha^{2\mu_-/\eta^2}}.$$

The probability that anti-predator action is taken, given that there is no predator, can be calculated from knowledge of the drift rates and variances by using a standard formula for the two-boundary case (e.g., see Taylor & Karlin, 1998) and taking limits.

In this case, it can be calculated directly by letting the death-rate of the process go to zero: $\phi(0)|_{\lambda=0} = \alpha^{-2\mu_-/\eta^2}$. However, we know by definition that the probability is $\alpha$, so we know that $2\mu_- = -\eta^2$ (easily confirmed by the calculations of Supplementary Material A). Therefore,

$$E(T_c|\mu_-, \text{AP action}) = \frac{\alpha ln(\alpha)}{\mu_-}.$$

$P(\text{AP action}|\mu_+)$ is simply the probability of the animal reaching the decision to take action before it is killed. This is the probability of survival, $R$, when $s = 1$.

Taking the result for $R$ from Supplementary Material B, we therefore have:

$$P(\text{AP action}|\mu_+) = \alpha^{\frac{-\mu_+ + \sqrt{\mu_+^2 + 2\theta\eta^2}}{\eta^2}}.$$

When a predator is present, the probability density function of time for the decision process to reach the boundary, irrespective of whether the animal has survived to that point, is provided by Karlin & Taylor (1975) as:

$$g(T) = \frac{-ln\alpha}{\sqrt{2\pi\eta^2 T^3}} exp\left(\frac{-(-ln\alpha - \mu_+ T)^2}{2\eta^2 T}\right).$$

Multiplying this by the probability of the animal surviving to that time, $e^{-\theta T}$, gives the probability density of the decision time in cases where a predator is present and the decision to take evasive action is made (before the focal animal is attacked).

The expectation can then be calculated in the usual manner:

$$E(T_c|\mu_+, \text{AP action}) = \int_0^\infty \frac{-ln\alpha}{\sqrt{2\pi\eta^2 T}} exp\left(\frac{-(ln\alpha + \mu_+ T)^2}{2\eta^2 T} - \theta T\right) dT$$

$$= \frac{-ln\alpha}{\sqrt{\mu_+^2 + 2\theta\eta^2}} \alpha^{\frac{-\mu_+ + \sqrt{\mu_+^2 + 2\theta\eta^2}}{\eta^2}}.$$

Letting $p'$ denote the probability of a predator being present given that the thalamus has not taken anti-predator action, we arrive at

$$E(T_c|\text{AP action}) = \frac{p' E(T_c|\mu_+, \text{AP action}) + (1-p')E(T_c|\mu_-, \text{AP action})}{p' P(\text{AP action}|\mu_+) + (1-p')P(\text{AP action}|\mu_-)}$$

$$= \frac{\frac{-p' ln(\alpha)}{\sqrt{\mu_+^2 + 2\theta\eta^2}} \alpha^{\frac{-\mu_+ + \sqrt{\mu_+^2 + 2\theta\eta^2}}{\eta^2}} + (1-p')\frac{\alpha ln(\alpha)}{\mu_-}}{p'\alpha^{\frac{-\mu_+ + \sqrt{\mu_+^2 + 2\theta\eta^2}}{\eta^2}} + (1-p')\alpha}$$

$$= \frac{-\mu_+ p' \alpha^{\frac{-3\mu_+ + \sqrt{\mu_+^2 + 2\theta\eta^2}}{\eta^2}} - (1-p')\sqrt{\mu_+^2 + 2\theta\eta^2}}{\mu_+ \sqrt{\mu_+^2 + 2\theta\eta^2}\left(p'\alpha^{\frac{-3\mu_+ + \sqrt{\mu_+^2 + 2\theta\eta^2}}{\eta^2}} + 1 - p'\right)} ln(\alpha) \quad \text{because } \mu_- = -\mu_+.$$

The total probability of anti-predator action occurring can be calculated as:

$$\begin{aligned} P(\text{AP Action}) &= P(\text{Thalamus acts}) \\ &\quad +(1 - P(\text{Thalamus acts}))\left(p' P(\text{Cortex acts}|\mu_+) + (1-p')P(\text{Cortex acts}|\mu_-)\right) \end{aligned}$$

$$= P(\text{Thalamus acts}) + (1 - P(\text{Thalamus acts}))\left(p'\alpha^{\frac{-\mu_+ + \sqrt{\mu_+^2 + 2\theta\eta^2}}{\eta^2}} + (1-p')\alpha\right).$$

## REFERENCES


Karlin, S. & Taylor, H. M. 1975 *First course in stochastic processes*, p. 363, 2nd edn. London, UK: Academic Press.

Taylor, H. M. & Karlin, S. 1998 *An introduction to stochastic modelling*, pp. 509–511, 3rd edn. San Diego, CA: Academic Press.