Supplementary Material B
Cortical threshold calculations

Let $f(z) = E(e^{-\theta T}|Z_0 = z)$ be the probability distribution of the probability of survival given that the decision will be reached at time T, with the log of the initial likelihood ratio being $Z_0 = z$.

Taking a small time slice, δt, from T, we let $T = \delta t + T'$, which gives: $e^{-\theta T} = e^{-\theta \delta t} e^{-\theta T'} \approx (1 - \theta \delta t) e^{-\theta T'}$.

Then, $f(z) \approx E((1 - \theta \delta t) e^{-\theta T'}|Z_0 = z)$

$= (1 - \theta \delta t) E(e^{-\theta T'}|Z_0 = z)$

$= (1 - \theta \delta t) E(f(z + \delta Z))$

where we take the movement δZ to have occurred during δt. Note that although the time slice is known, the amount of movement in that time is a random variable.

Using a Taylor expansion, we obtain

$$f(z) \approx (1 - \theta \delta t) \left(f(z) + E(\delta Z) f'(z) + \frac{E((\delta Z)^2)}{2} f''(z) \right).$$

Discarding terms smaller than δt:

$$0 \approx E(\delta Z) f'(z) + \frac{E((\delta Z)^2)}{2} f''(z) - \theta \delta t f(z). \tag{1}$$

In this case, the movement in position with each timestep, δt, is normally distributed according to $N(\mu \delta t, \eta^2 \delta t)$, where μ and η are calculated as shown in Supplementary Material A (with $\mu = \mu_+$ as there is a predator present). From the definition of the computational formula for variance, $Var(\delta Z) = E((\delta Z)^2) - (E(\delta Z))^2$, so $\eta^2 \delta t = E((\delta Z)^2) - (\mu \delta t)^2$. Substituting in equation (1), this gives:

$$0 \approx E(\delta Z) f'(z) + \frac{(\mu \delta t)^2 + \eta^2 \delta t}{2} f''(z) - \theta \delta t f(z)$$

The $(\delta t)^2$ term can be discarded (in-keeping with the above). We also know that $E(\delta Z) = \mu \delta t$. Substituting, dividing through by δt and rearranging, we obtain:

$$0 \approx -\theta f(z) + \mu f'(z) + \frac{\eta^2}{2} f''(z)$$

As δt goes to zero and the information gain becomes continuous, the equation becomes exact.

Solving for $f(z)$, the characteristic equation $0 = -\theta + \mu k + \frac{\eta^2}{2} k^2$ has solutions $k_1 = \frac{-\mu + \sqrt{\mu^2 + 2\theta\eta^2}}{\eta^2}$, $k_2 = \frac{-\mu - \sqrt{\mu^2 + 2\theta\eta^2}}{\eta^2}$.

The general solution is then given by:

$$f(z) = K_1 e^{k_1 z} + K_2 e^{k_2 z}. \tag{2}$$

We define A to be the ratio of likelihoods at (or above) which the animal should act as though a predator is present. i.e., for a sequence of signals, $(x_1, x_2, ..., x_k)$, the animal should take evasive action if and only if $\frac{f_1(x_1)f_1(x_2)...f_1(x_k)}{f_0(x_1)f_0(x_2)...f_0(x_k)} \geq A$. Then we know that $f(lnA) = 1$ and $f(z) \to 0$ as $z \to -\infty$.

Substituting the second of these boundary conditions into equation (2) and noting that k_2 is negative, we find that $K_2 = 0$. Substituting the first boundary condition, we obtain $K_1 = e^{-k_1 lnA} = A^{-k_1}$.

Therefore, knowing that our process will start at $Z_0 = 0$, the probability of survival, R, is given by:

$$R = sf(0) = s(K_1 + K_2) = sA^{-k_1}. \tag{3}$$

As we are dealing with a continuous process, any decision to take anti-predator action will be reached at the point of hitting the boundary (as opposed to overshooting it), so rather than having to approximate A, the formula supplied by Wald (1945) is exact: $A = \frac{1-\beta}{\alpha}$, where α is the probability of a false alarm (taking evasive action when no predator is present) and β is the probability of a miss (deciding that no predator is present and ignoring any further signals). In this case, as the probability of a miss is zero, we have $A = \frac{1}{\alpha}$.

Putting this into equation (3), we obtain: $R = s\alpha^{k_1}$, so $\frac{dR}{d\alpha} = sk_1\alpha^{k_1-1}$.

Our original ODE (in the main paper) was:

$$(1-p)c = p(v_f - c)\frac{dR}{d\alpha}$$

Substituting for $\frac{dR}{d\alpha}$, we obtain:

$$\alpha^{k_1-1} = \frac{(1-p)c}{p(v_f - c)k_1 s}.$$